Optimizing Harvest Control Rules In the Presence of Natural Variability and Parameter Uncertainty

Grant G. Thompson

NMFS, Alaska Fisheries Science Center, 7600 Sand Point Way NE., Seattle, WA 98115-0070.

E-mail address: Grant.Thompson@noaa.gov

Abstract.- Classical one-parameter harvest policies (such as those based on maintaining a constant optimal catch, constant optimal fishing mortality rate, or constant optimal escapement) and full optimal control solutions (such as those generated through stochastic dynamic programming) represent two ends of a spectrum of possible *harvest control rules*. The classical one-parameter policies have little flexibility and may be substantially sub-optimal, but are easy to describe. True optimal control policies, on the other hand, are completely flexible and fully optimal, but they can be inaccessible. As a compromise between the classical one-parameter policies and a full optimal control solution, several authors have suggested that fisheries be managed by specifying the functional form of a control rule *a priori* and then choosing values for one or more of the parameters so as to maximize a management objective function. The purpose of this paper is to gain an increased understanding of how such harvest control rules can be used to address the problem of optimal fishery management. This is undertaken in three stages, proceeding in order of increasing complexity. In Stage 1, the analysis assumes that population dynamics are completely deterministic and that the values of all biological parameters are known with certainty. Here, the focus of optimization is on maximum sustainable yield (MSY). In Stage 2, the analysis is generalized to the case in which natural (stochastic) variability is present but the values of all biological parameters are still known with certainty. Here, the focus of optimization is on a stochastic analogue of MSY, with attention paid also to tradeoffs between long-term average yield and the level of variability around that average. In Stage 3, the analysis is further generalized to the case in which natural variability is present and the values of biological parameters are uncertain. Here, the focus of optimization is on decision-theoretic analogues of MSY, with attention paid to the various features desired under a precautionary approach. At each stage of development, a general treatment of the problem is attempted, followed by a specific example. Some implications of alternative control rules with respect to the special problem of rebuilding a depleted stock are also given for the first two stages.

Introduction

Background

Development of the Theory

Design of optimal harvest strategies has been a major emphasis of fisheries science throughout most of this century. Early on (e.g., Russell 1931, Hjort et al. 1933), efforts focused on identification of a "constant catch" policy; that is, a single, time-invariant *catch* which could be taken year after year. Soon, though, investigators (e.g., Thompson and Bell 1934, Graham 1935) began focusing on identification of a "constant fishing mortality" policy; that is, a single, time-invariant *fishing mortality rate* which could be applied year after year. Some twenty years later, Ricker (1958) focused on use of a "constant escapement" policy; that is, a single, time-invariant *escapement* which would remain in the stock following each year's harvest. Each of these strategies was developed in the hope of obtaining the maximum sustainable yield (MSY) from the fishery, although the definitions of this term have sometimes been unclear or inconsistent. Since these early investigations, many studies have compared the policies of constant catch, constant fishing mortality, and constant escapement. One of the earliest and most thorough comparative evaluations of these three policies was conducted by Reed (1978). Other comparisons of two or more of these policies have been made by Tautz et al. (1969), Gatto and Rinaldi (1976), Beddington and May (1977), May et al. (1978), Hilborn (1979), Deriso (1985), Hilborn and Walters (1992), Frederick and Peterman (1995), and Steinshamn (1998).

Although the focus of each of the above policies is distinct from the others, they share the characteristic that each distills the optimal harvest problem into a single (albeit different) parameter. More complicated policies have also been explored. At about the same time that Ricker was considering the merits of a constant escapement policy, Scott (1955) noted that a truly optimal management strategy would not necessarily be describable in terms of a single parameter. Rather, Scott argued that the optimal harvest should be conceptualized as an entire time series of future catches, *each* of which is chosen in the context of all the others so that the overall benefits from the fishery are maximized. It was not until the 1970s, however, that formal analyses of such a policy were successfully undertaken. These analyses typically involved application of the Pontryagin maximum principle (Pontryagin et al. 1962). Such treatments include those given by Quirk and Smith (1970), Plourde (1970, 1971), and Cliff and Vincent (1973), but Clark's

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(1973, 1976) solution of the simple, deterministic model attributed to Gordon (1954) and Schaefer (1954) is probably the best remembered of this group of studies. Somewhat ironically, it turned out that the strategy which resulted in full optimization of the Gordon-Schaefer model was simply a particular type of constant escapement policy.

While Clark's (1973, 1976) use of the Pontryagin maximum principle in solving simple fishery models was instrumental in bringing an "optimal control" perspective to the design of harvest strategies, application of the maximum principle to more complicated models involving natural variability or parameter uncertainty has not been particularly successful (for an exception, see Gleit 1978). Instead, other techniques such as stochastic dynamic programming have been employed to identify optimal control strategies. Examples are given by Reed (1974, 1979), Walters (1975), Hilborn (1976), Getz (1979), Dudley and Waugh (1980), Mendelssohn (1980, 1982), Charles (1983), Mangel (1985), Hightower and Grossman (1987), Horwood et al. (1990), and Horwood (1991, 1993).

Unfortunately, full optimal control solutions are at best computationally intensive, and at worst completely opaque. Describing such solutions, Horwood (1993, p. 341) states,

> "For deterministic problems they are costly in time, but more importantly do not allow the construction of a general management control. The stochastic laws cannot be derived."

The classical one-parameter policies and the full optimal control solutions thus represent two ends of a spectrum of possible *harvest control rules* ("feedback control laws" in the terminology of Clark 1976): The classical one-parameter policies have little flexibility and may be substantially sub-optimal, but they are easy to describe. True optimal control policies, on the other hand, are completely flexible and fully optimal, but they can be inaccessible. As a compromise between the classical one-parameter policies and a full optimal control solution, several authors have suggested that fisheries be managed by specifying the functional form of a control rule *a priori* and then choosing values for one or more of the parameters so as to maximize a management objective function. Walters and Hilborn (1978) called this approach "fixed form optimization," and described it as follows (p. 167):

> "There are two basic steps in the development of a fixed-form optimization. The first is to find an algebraic form of the control function. Intuition, common sense, etc. can often be used to guess at a reasonable form.... The second step in fixed-form optimization is to find the optimal values of the control parameters."

Larkin and Ricker (1964) were among the first to suggest such an approach. Specifically, their suggestion was to prohibit fishing whenever escapement failed to reach a specified level but to allow fishing at a constant rate whenever escapement exceeded the specified level. This 2-parameter policy has also been explored by Aron (1979), Quinn et al. (1990), and Zheng et al. (1993). Other multi-parameter forms for possible control rules were subsequently suggested or evaluated by Allen (1973), Walters and Hilborn (1978), Shepherd (1981), Ruppert et al. (1984, 1985), Hilborn (1985), Getz et al. (1987), Hightower and Lenarz (1989), Hightower (1990), and Engen et al. (1997).

Implementation of the Theory

As is often the case when moving from "theory" to "application" in fisheries management, it has proven easier to evaluate harvest control rules in the literature than to implement them in practice. However, significant progress has been made in the past decade. In the United States, a 2-parameter control rule (based on the functional form suggested by Shepherd 1981) was adopted for management of groundfish off Alaska in 1990. In an official review of overfishing definitions used in the United States, Rosenberg et al. (1994) recommended that a control rule approach be used "whenever it is practical," and suggested a possible functional form. Based in part on this suggestion, the Alaska groundfish control rule was later modified to a 3-parameter form (U.S. National Marine Fisheries Service 1996). More recently, the Northwest Atlantic Fishery Organization (Serchuk et al. 1997) and the International Council for the Exploration of the Sea (1997) have explored the use of harvest control rules. Finally, the U.S. Government issued a set of "National Standard Guidelines" in 1998 which assigned a fundamental role to harvest control rules (U.S. Department of Commerce, 1998).

Harvest Control Rules and the Precautionary Approach

Much of the current interest in harvest control rules stems from a perception that they can play an important role in implementing a "precautionary approach" to fisheries management. At the international level, calls for adoption of such an approach have been featured in several agreements developed under the auspices of the United Nations, including the Code of Conduct for Responsible Fisheries prepared by the United Nations Food and Agriculture Organization (FAO), the FAO Technical Consultation on the Precautionary Approach to Capture Fisheries, the Rio Declaration of the United Nations Conference on Environment and Development, and the United Nations Convention on the Law of the Sea Relating to the Conservation and Management of Straddling Stocks and Highly Migratory Fish Stocks (the "Straddling Stocks Agreement"). For example, Annex II of the Straddling Stocks Agreement (United Nations 1995) includes the following provisions:

> "Two types of precautionary reference points should be used: conservation, or limit, reference points and management, or target, reference points";
>
> "fishery management strategies shall ensure that the risk of exceeding limit reference points is very low"; and
>
> "the fishing mortality rate which generates maximum sustainable yield should be regarded as a minimum standard for limit reference points."

In the U.S., the National Standard Guidelines (U.S. Department of Commerce 1998) also encourage the use of a precautionary approach with the following features:

> "Target reference points ... should be set safely below limit reference points...";
>
> "a stock ... that is below the size that would produce MSY should be harvested at a lower rate or level of fishing mortality than if the stock ... were above the size that would produce MSY"; and
>
> "criteria used to set target catch levels should be explicitly risk averse, so that greater uncertainty regarding the status or productive capacity of a stock or stock complex corresponds to greater caution in setting target catch levels."

A more detailed description of the historical development of the precautionary approach has been given by Thompson and Mace (1997).

Purpose and Outline

The purpose of this paper is to gain an increased understanding of how harvest control rules can be used to address the problem of optimal fishery management. This will be undertaken in three stages, proceeding in order of increasing complexity. In Stage 1, the analysis will assume that population dynamics are completely deterministic and that the values of all biological parameters are known with certainty. Here, the focus of optimization will be on MSY. In Stage 2, the analysis will be generalized to the case in which natural (stochastic) variability is present but the values of all biological parameters are still known with certainty. Here, the focus of optimization will be on a stochastic analogue of MSY, with attention paid also to tradeoffs between long-term average yield and the level of variability around that average. In Stage 3, the analysis will be further generalized to the case in which natural variability is present and the values of biological parameters are uncertain. Here, the focus of optimization will be on decision-theoretic analogues of MSY, with attention paid to the various features desired under a precautionary approach (U.S. Department of Commerce 1998). At each stage of development, a general treatment of the problem will be attempted, followed by a specific example. Some implications of alternative control rules with respect to the special problem of rebuilding a depleted stock will also be given for the first two stages.

The outline of the remainder of the paper is thus as follows:

Stage 1: Determinism Under Known Parameter Values
- Dynamics
- Solution
- Rebuilding
- Optimization

Stage 2: Incorporating Natural Variability
- Dynamics
- Solution
- Rebuilding
- Optimization

Stage 3: Incorporating Parameter Uncertainty

Discussion

Table 1 lists the symbols used in the remainder of the paper. A definitional change regarding one parameter will prove helpful in moving from Stage 2 to Stage 3. This is addressed in the text.

Stage 1: Determinism Under Known Parameter Values

Dynamics

In General

In the absence of both natural variability ("process error") and fishing, let the dynamics of stock size x be modeled in continuous time t as the ordinary differential equation

$$\frac{dx}{dt} = f(x|\boldsymbol{\chi}), \qquad (1)$$

where f is a function and $\boldsymbol{\chi}$ is a parameter vector of length m.

Table 1.- Symbols used in this paper.

Variables		**Means of a Random Variable**	
t	time	A	arithmetic mean
x	stock size	G	geometric mean
y	yield	H	harmonic mean
Elementary Parameters		**Parameters of Statistical Distributions**	
a	Gompertz growth parameter	α	first beta shape parameter
b	Gompertz scale parameter	β	second beta shape parameter
c	control rule intercept parameter	η	inverse Gaussian scale parameter
d	control rule slope parameter	θ	inverse Gaussian shape parameter
s	process error scale parameter	μ	ln(lognormal scale parameter)
z	objective function weight parameter	σ	lognormal shape parameter
Functions of Stock Size Only		**Parameter Vectors**	
f	function describing deterministic dynamics	$\boldsymbol{\chi}$	vector of parameters used in f
g	function describing process error scale	$\boldsymbol{\psi}$	vector of parameters used in g
h	function describing harvest control rule	$\boldsymbol{\omega}$	vector of parameters used in h
Functions of Stock Size or Other Variables		**Constants**	
p	probability density function	e	Napier's constant (2.7183...)
q	objective function	m	dimension of $\boldsymbol{\chi}$
r	normalized process error function	n	dimension of $\boldsymbol{\psi}$
Composite Parameters		**Functions of Means**	
u	ratio of a to $a+d$	k_a	function of H_a/A_a and w
v	ratio of s^2 to $2a$	k_b	function of H_b/A_b and A_u
w	ratio of d to a (Stage 2) or d to A_a (Stage 3)	k_v	function of H_v/A_v, A_u, and A_v

Next, consider a function which uses a parameter vector $\boldsymbol{\omega}$ to map stock size x into an instantaneous harvest (fishing) mortality rate h. Such a function constitutes a "harvest control rule." The purpose of a harvest control rule is to associate a reference fishing mortality rate (either a target or a limit) with each possible stock size. For any harvest control rule, yield y at time t will be the product of x at time t and h, where h itself is a function of x. The time derivative of stock size then becomes

$$\frac{\mathrm{d}\,x}{\mathrm{d}\,t} = f\,(x\,|\boldsymbol{\chi}\,)\;-\;h(x\,|\boldsymbol{\omega})\,x\,. \tag{2}$$

For Example

When fishing is absent, the Gompertz (1825) biomass dynamic model can be viewed as an example of Equation (1), with $\boldsymbol{\chi}=(a,b)^{\mathrm{T}}$:

$$\frac{\mathrm{d}\,x}{\mathrm{d}\,t} = a\,x\left(1\,-\,\ln\left(\frac{x}{b}\right)\right), \tag{3}$$

where a is a growth rate and b is a scale parameter.

The simplest case of a harvest control rule occurs when $\boldsymbol{\omega}$ is a scalar c and h is a constant (i.e., $h(x) = c$). When this control rule is assumed, the Gompertz model becomes the Gompertz-Fox model (Fox 1970):

$$\frac{\mathrm{d}\,x}{\mathrm{d}\,t} = a\,x\left(1\,-\,\ln\left(\frac{x}{b}\right)\right) - c\,x\,.$$

More complicated rules can be imagined as the dimension of $\boldsymbol{\omega}$ increases. For example, if $\boldsymbol{\omega}$ consists of a pair of control parameters c and d, some possible harvest control rules include the hyperbolic form $h(x) = c - d/x$, the square-root form $h(x) = c + d\sqrt{x}$, the linear form $h(x) = c + d\,x$, and the logarithmic form $h(x) = c + d\ln(x)$. In any of these examples, setting d=0 gives the one-parameter control rule $h(x) = c$. The hyperbolic form was considered (after translating to (x,y)-space; that is, $y(x) = c\,x - d$) by Hilborn (1985), Hightower and Lenarz (1989), and Engen et al. (1997). In addition, it conforms to a special case of the three-parameter control rule considered by Ruppert et al. (1984, 1985) and Hightower and Lenarz (1989). (The square-root and linear forms, both with c=0, also correspond to special cases of this three-parameter control rule.) The linear form was considered by Hightower (1990). If the underlying stock dynamics are governed by a model of the form suggested by Graham (1935) and Schaefer (1954), a linear control rule would be a natural choice in that such a control rule would not change the stock dynamics in any qualitative way. Given stock dynamics of the form suggested by Gompertz (1825), however, the logarithmic form is the natural choice, as shown below. Assuming Gompertz dynamics and a logarithmic control rule, the time derivative of stock size becomes

$$\frac{dx}{dt} = a\,x\left(1 - \ln\left(\frac{x}{b}\right)\right) - h(x)\,x$$

$$= a\,x\left(1 - \ln\left(\frac{x}{b}\right)\right) - \left(c + d\ln(x)\right)x \qquad (4)$$

$$= (a + d)\,x\left(\frac{a(1+\ln(b)) - c}{a + d} - \ln(x)\right)$$

Comparing the above with Equation (3) shows that use of a logarithmic control rule does not alter the underlying stock dynamics in any qualitative way. The Gompertz-Fox model corresponds to the special case of the above in which d=0. Examples of logarithmic control rules are shown in Figure 1.

Solution

In General

The time trajectory of stock size will generally be of the form $x(\chi, \omega, x_0, t)$, where x_0 represents an initial condition and t is measured with respect to an initial time t_0=0. In the limit as t approaches infinity, the trajectory will converge to the equilibrium value $x^*(\chi, \omega)$, assuming such an equilibrium exists.

For Example

The equilibrium stock size implied by Equation (4) is given by

$$x^*(a, b, c, d) = e^{\frac{a(1+\ln(b)) - c}{a + d}}$$

and the time trajectory is given by

$$x(a, b, c, d, x_0, t) =$$

$$x^*(a, b, c, d)\left(\frac{x_0}{x^*(a, b, c, d)}\right)^{e^{-(a+d)t}} . \qquad (5)$$

For the special case c=d=0 (i.e., no fishing), the equilibrium stock size is simply be.

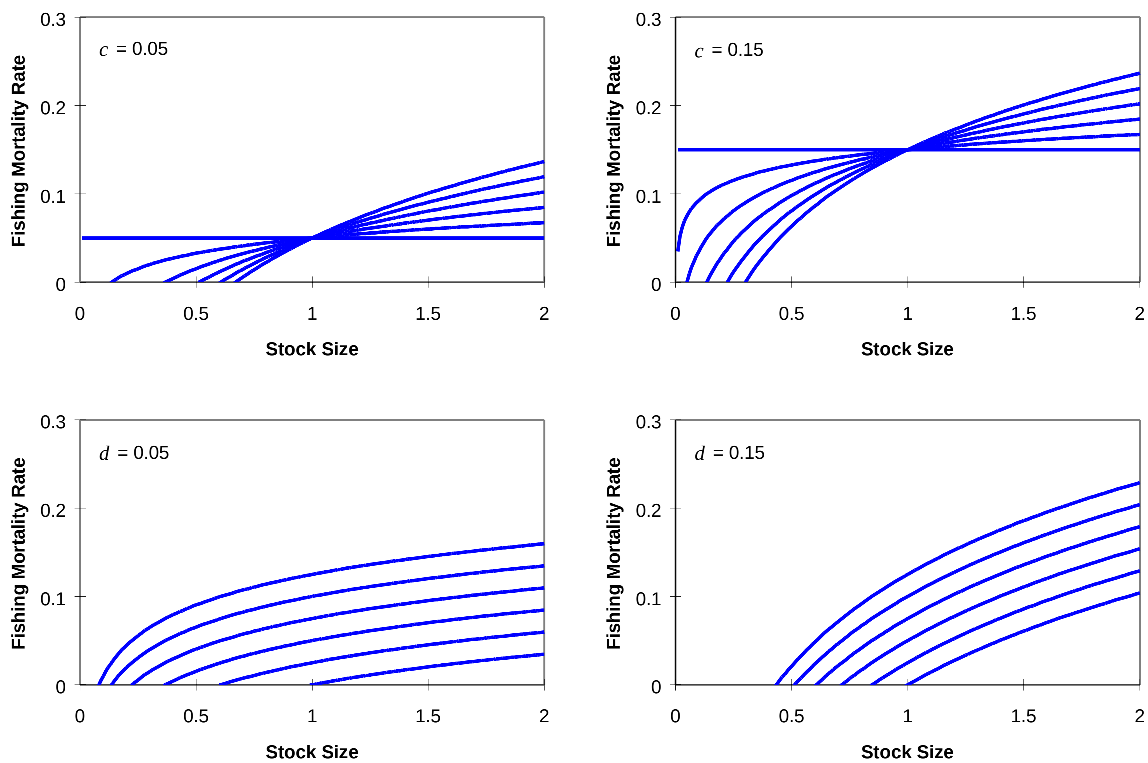

Figure 1. Example control rules. In each of the upper panels, the slope of the control rule increases directly with d. In each of the bottom panels, the height of the control rule increases directly with c.

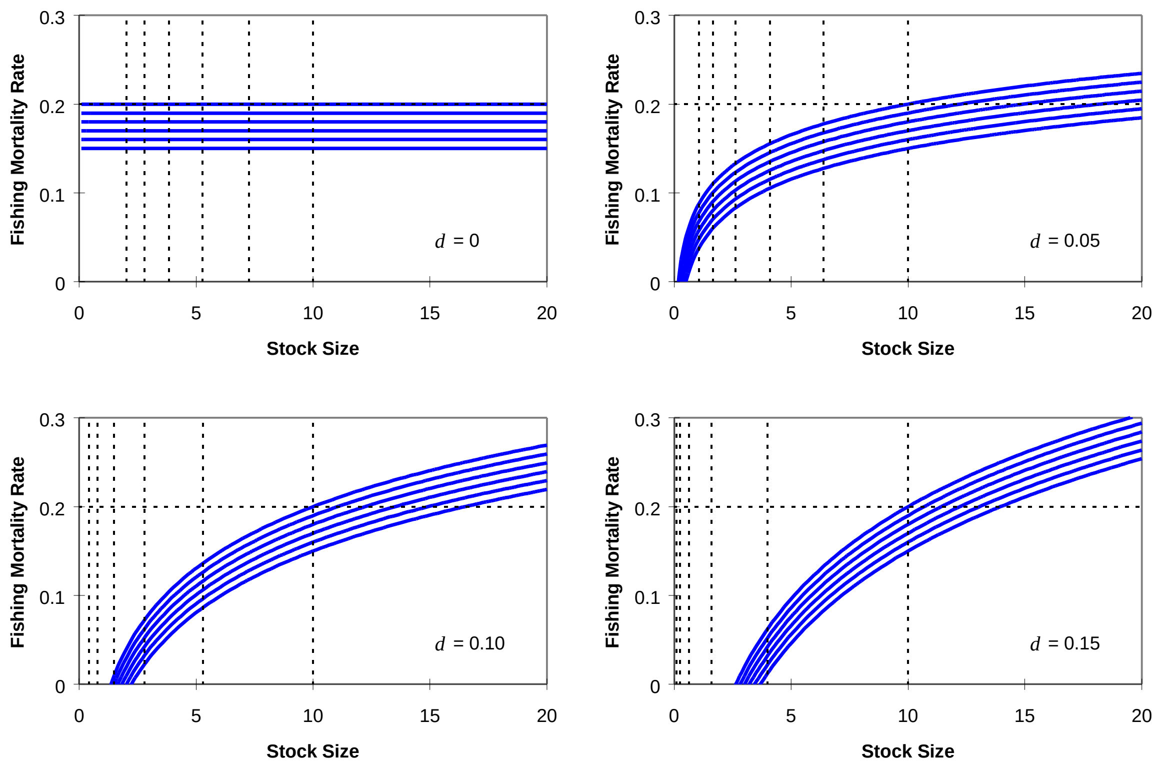

Figure 2. Example control rules (solid curves) and associated thresholds (vertical dotted lines). In each panel, as c decreases, the control rule moves down and the threshold moves left.

Rebuilding

In General

In practice, fish stocks are often observed to be at levels of abundance well below those considered to be optimal, or even safe. In such situations, fisheries scientists are frequently asked to estimate how much time will elapse before the stock rebuilds to some reference level, contingent upon implementation of a specified harvest policy in the interim. More proactively, the "rebuilding" question may be phrased this way: How low can a stock's level of abundance fall and still rebuild to a size x_{reb} within a time period t_{reb} under a specified harvest policy? This is not the only issue which a rebuilding plan might logically address (e.g., Powers 1996), but it is a central one (e.g., as implied by U.S. Department of Commerce 1998). The answer is obtained by solving the equation $x(\chi, \omega, x_{thr}, t_{reb}) = x_{reb}$ for the "threshold" stock size x_{thr}.

For Example

Setting Equation (5) equal to x_{reb}, setting $t=t_{reb}$, and solving for x_0 (relabeled x_{thr}) gives

$$x_{thr} = x^*(a,b,c,d)\left(\frac{x_{reb}}{x^*(a,b,c,d)}\right)^{e^{(a+d)t_{reb}}}.$$

In the special case where $x_{reb}=b$, the above simplifies to

$$x_{thr} = b\exp\left[-\left(\frac{a-c-d\ln(b)}{a+d}\right)\left(e^{(a+d)t_{reb}}-1\right)\right] \tag{6}$$

Examples of logarithmic harvest control rules and their corresponding stock size thresholds for parameter values a=0.2, b=10, x_{reb}=10, and t_{reb}=10 are shown in Figure 2. In each of Figure 2's four panels, the uppermost control rule passes through the point (b,a), indicated by the intersection of the horizontal dotted line and the rightmost vertical dotted line. Whenever $x_{reb}=b$, any harvest control rule passing through the point (b,a), meaning any control rule in which $h(b)=a$, will always have a threshold stock size equal to b. Furthermore, control rules in which $h(b)<a$ will always have a threshold stock size less than b in such cases (i.e., whenever $x_{reb}=b$).

Optimization

In General

Sustainable (equilibrium) yield can be viewed as a function of the parameter vectors $\boldsymbol{\chi}$ and $\boldsymbol{\omega}$. To keep things relatively simple throughout the remainder of the paper, let the control parameter c correspond to the ith element of $\boldsymbol{\omega}$ and let $\boldsymbol{\omega}_{(i)}$ denote the vector $\boldsymbol{\omega}$ with the ith element (i.e., c) removed, and let sustainable yield be written $y^*(c\,|\,\boldsymbol{\chi},\boldsymbol{\omega}_{(i)})$ to emphasize the dependence of sustainable yield on c. Then, MSY is achieved conditionally on $\boldsymbol{\chi}$ and $\boldsymbol{\omega}_{(i)}$ by finding the value $c_{\mathrm{MSY}}(\boldsymbol{\chi},\boldsymbol{\omega}_{(i)})$ such that the following equation is satisfied:

$$\frac{\mathrm{d}\,y^*(c\,|\,\boldsymbol{\chi},\boldsymbol{\omega}_{(i)})}{\mathrm{d}\,c}\,\Big|_{c=\,c_{\mathrm{MSY}}(\boldsymbol{\chi},\boldsymbol{\omega}_{(i)})}=0.$$

For Example

The sustainable yield corresponding to Equation (4) is given by substituting $x^*(a,b,c,d)$ into the logarithmic control rule, giving

$$y^*(c\,|\,a,b,d)=\left(c+d\ln(x^*(a,b,c,d))\right)x^*(a,b,c,d)$$

$$=\left[c+d\left(\frac{a(1+\ln(b))-c}{a+d}\right)\right]\exp\left(\frac{a(1+\ln(b))-c}{a+d}\right).$$

Given a value of either of the control parameters c and d, it is possible to solve for the value of the other so that sustainable yield is maximized. For example, if the solution is conditioned on control parameter d, MSY is obtained by setting

$$c_{\mathrm{MSY}}(a,b,d)=a-d\ln(b).$$

Thus, an "MSY control rule" for this model is any rule of the form

$$h(x\,|\,a,b,d)=c_{\mathrm{MSY}}(a,b,d)+d\ln(x)$$

(cf. U.S. Department of Commerce 1998). In Figure 2, for example, the uppermost curve in each panel is an MSY control rule. The Gompertz-Fox model corresponds to the special case where d=0, giving $c_{\mathrm{MSY}}(a,b,0)=a$. Changing the value of d allows the MSY control rule to be viewed as a continuum extending from a constant fishing mortality policy at one end (d=0) to a constant escapement policy at the other end (in the limit as d approaches infinity).

For any MSY control rule of the above form, equilibrium stock size is equal to b. MSY itself is equal to the product ab, and is thus independent of d.

Stage 2: Incorporating Natural Variability

Dynamics

In General

Equation (2) can be generalized to a stochastic differential equation incorporating random natural variability as follows:

$$\frac{\mathrm{d}\,x}{\mathrm{d}\,t}=f(x\,|\boldsymbol{\chi})+g(x\,|\boldsymbol{\psi})\,r(t)-h(x\,|\boldsymbol{\omega})\,x, \quad (7)$$

where $r(t)$ is a standard white noise process and g is a function of x, with parameter vector $\boldsymbol{\psi}$ of length n, that scales the intensity of the noise. It should be noted that the interpretation of stochastic differential equations given by Stratonovich (1963) is used here (e.g., Ricciardi 1977).

For Example

Natural variability can be added to the deterministic Gompertz model with a logarithmic harvest control rule by setting $\boldsymbol{\psi}=s$, $g(x\,|\boldsymbol{\psi})=sx$ and recasting the time derivative as a stochastic differential equation of the form

$$\frac{\mathrm{d}\,x}{\mathrm{d}\,t}=a\,x\left(1-\ln\left(\frac{x}{b}\right)\right)+s\,x\,r(t)-\left(c+d\ln(x)\right)x. \quad (8)$$

Solution

In General

Broadly speaking, stock size at time t could potentially range anywhere from zero to arbitrarily large, though some stock sizes are more probable than others. Given an initial condition x_0, this fact can be modeled as a pdf with parameter vector $(\boldsymbol{\chi}^{\mathrm{T}}, \boldsymbol{\psi}^{\mathrm{T}}, \boldsymbol{\omega}^{\mathrm{T}}, x_0, t)^{\mathrm{T}}$. More precisely, the probability that stock size falls between x_1 and x_2 at time t may be written in terms of the "transition distribution" $p_x(x\,|\,\boldsymbol{\chi},\boldsymbol{\psi},\boldsymbol{\omega},x_0,t)$ as follows:

$$Pr\,(x_1\le x(t)\le x_2)=\int_{x_1}^{x_2}p_x(x\,|\,\boldsymbol{\chi},\boldsymbol{\psi},\boldsymbol{\omega},x_0,t)\,\mathrm{d}\,x.$$

In the limit as t approaches infinity, p_x (if it still exists) describes the "stationary distribution" of x. The stationary distribution can be written $p_x^*(x\,|\,\boldsymbol{\chi},\boldsymbol{\psi},\boldsymbol{\omega})$.

For Example

Using a different parametrization, the solution to Equation (8) was considered for the special case c=d=0 (i.e., no harvesting) by Capocelli and Ricciardi (1974). The less restricted case d=0 (with c arbitrary) was considered by Thompson (1998). When no restrictions are placed on either c or d, the stationary distribution of stock size is lognormal, specifically,

$$p_x^*(x|a,b,s,c,d) = \sqrt{\frac{1}{2\pi}}\left(\frac{1}{\sigma_x^*(a,s,d)\,x}\right)\times \exp\left(-\left(\frac{1}{2}\right)\left(\frac{\ln(x)-\mu_x^*(a,b,c,d)}{\sigma_x^*(a,s,d)}\right)^2\right),$$

where

$$\mu_x^*(a,b,c,d) = \frac{a(1+\ln(b))-c}{a+d} = \ln\left(x^*(a,b,c,d)\right)$$

and

$$\sigma_x^*(a,s,d) = \frac{s}{\sqrt{2(a+d)}}.$$

Similarly, the transition distribution of stock size at time t is also lognormal, specifically,

$$p_x(x|a,b,s,c,d,x_0,t) = \sqrt{\frac{1}{2\pi}}\left(\frac{1}{\sigma_x(a,s,d,t)\,x}\right)\times \exp\left(-\left(\frac{1}{2}\right)\left(\frac{\ln(x)-\mu_x(a,b,c,d,x_0,t)}{\sigma_x(a,s,d,t)}\right)^2\right) \quad (9)$$

where

$$\mu_x^*(a,b,c,d,x_0,t) = e^{-(a+d)t}\ln(x_0) + \left(1-e^{-(a+d)t}\right)\mu_x^*(a,b,c,d)$$

and

$$\sigma_x(a,s,d,t) = \sqrt{1-e^{-2(a+d)t}}\;\sigma_x^*(a,s,d).$$

<u>Rebuilding</u>

In General

In the presence of natural variability, discussion of rebuilding trajectories can become much more complicated than in the deterministic case. Because an infinite number of rebuilding trajectories is possible in the stochastic case, rebuilding is typically described using some sort of summary statistic. For example, the following equation could be solved for x_{thr} after substituting some desired probability of successful rebuilding (e.g., 50%) for the left-hand side:

$$\Pr(x_{reb} \le x(t_{reb}) \le \infty) = \int_{x_{reb}}^{\infty} p_x(x|\chi,\psi,\omega,x_{thr},t_{reb})\,dx.$$

Alternatively, the solution could be expressed in terms of expected values of x (or some transformation thereof) at time $t=t_{reb}$, for example, by equating x_{reb} with the arithmetic mean or geometric mean of x at time $t=t_{reb}$.

For Example

Unlike the general case, a fortunate property of the model used here is that consideration of rebuilding schedules in the presence of natural variability need not be any more complicated than in the deterministic situation described in Stage 1, depending on the choice of summary statistic. Because the geometric mean of the transition distribution [Equation (9)] is identical to the deterministic solution of the time trajectory [Equation (5)], and because the lognormal form of the transition distribution implies that the geometric mean is equal to the median, using either the geometric mean or a 50% probability of exceeding x_{reb} to compute the threshold stock size x_{thr} gives the same result as in the deterministic case [Equation (6)].

<u>Optimization</u>

In General

The conditional arithmetic mean of the stationary distribution of yield is defined as

$$A_y(\chi,\psi,\omega) = \int_0^{\infty} y(x|\omega)\,p_x^*(x|\chi,\psi,\omega)\,dx.$$

The dependence of the conditional arithmetic mean on a particular control parameter can be emphasized by rewriting $A_y(\chi,\psi,\omega)$ as $A_y(c|\chi,\psi,\omega_{(i)})$, following the Stage 1 convention in which control parameter c corresponds to the ith element of ω. Then, this quantity can be maximized with respect to control parameter c by differentiating, setting the resulting expression equal to zero, and solving with respect to c. Maximizing $A_y(c|\chi,\psi,\omega_{(i)})$ with respect to c gives the control parameter value associated with maximum expected stationary yield (MESY):

$$\frac{d\,A_y(c|\chi,\psi,\omega_{(i)})}{d\,c}\Big|_{c=\tilde{c}_{\mathrm{MESY}}(\chi,\psi,\omega_{(i)})} = 0,$$

where use of the "~" symbol is intended to denote that the maximization is conducted with respect to the conditional mean (alternative maximizations will be described later).

Much of the literature concerning optimal harvest strategies in the presence of natural variability deals with tradeoffs between the magnitude of yield on average and the variability about that average. In the context of comparisons between the classical one-parameter harvest policies, such tradeoffs have been considered by Ricker (1958), Larkin and Ricker (1964), Gatto and Rinaldi (1976), Beddington and May (1977), May et al. (1978), Reed (1978), Hilborn (1979), Hilborn and

Walters (1992), Frederick and Peterman (1997), and Steinshamn (1998). In the context of optimal control policies, they have been considered by Walters (1975), Mendelssohn (1980), and Horwood et al. (1990). In the context of fixed-form control rules, they have been considered by Allen (1973), Aron (1979), Hilborn (1985), Ruppert et al. (1985), Getz et al. (1987), Hightower and Lenarz (1989), Hightower (1990), Quinn et al. (1990), Zheng et al. (1993), and Engen et al. (1997).

One way to characterize the variability of yield on a scale equivalent to that of the arithmetic mean is by the standard deviation. If c is set equal to $\tilde{c}_{\text{MESY}}(\boldsymbol{\chi}, \boldsymbol{\psi}, \boldsymbol{\omega}_{(i)})$, both the arithmetic mean and standard deviation of the stationary distribution of y will be functions of $\boldsymbol{\chi}$, $\boldsymbol{\psi}$, and $\boldsymbol{\omega}_{(i)}$, meaning that tradeoffs between the arithmetic mean and standard deviation can be viewed as a function of the control parameter sub-vector $\boldsymbol{\omega}_{(i)}$ for given values of $\boldsymbol{\chi}$ and $\boldsymbol{\psi}$.

For Example

By defining a natural variability level

$$v \equiv \sigma_x^*(a,s,0)^2 = \frac{s^2}{2a}$$

(i.e., by defining v as the variance of the stationary distribution of log stock size when d=0), the equations for many quantities of interest in the example model can be simplified considerably. Thus, wherever s appears as a parameter in a particular equation, it can be replaced with the quantity $\sqrt{2av}$, and whenever s appears as a function argument in a particular equation, it can be replaced with the parameter v. Similarly, by defining a scaled control parameter

$$w \equiv \frac{d}{a}$$

(i.e., by viewing the control parameter d relative to a rather than in absolute terms) and reparametrizing accordingly, it turns out that a appears only as a constant of proportionality in many (but not all) quantities of interest in this model. Thus, wherever d appears as a parameter in a particular equation, it can be replaced with the quantity wa, and wherever d appears as a function argument in a particular equation, it can be replaced with the parameter w.

With these composite parameters, the conditional arithmetic mean of the stationary distribution of stock size x can be written as

$$A_x(a,b,v,c,w) = \int_0^\infty x\ p_x^*(x|a,b,v,c,w)\ \mathrm{d}x$$

$$= \exp\left[\left(\frac{1}{1+w}\right)\left(\frac{c}{wa}+1+\ln(b)+\frac{v}{2}\right)-\frac{c}{wa}\right],$$

and the conditional arithmetic mean of the stationary distribution of yield y can be written as

$$A_y(a,b,v,c,w) = \int_0^\infty \big(c + d\ln(x)\big)\ x\ p_x^*(x|a,b,v,c,w)\ \mathrm{d}x$$

$$= wa\left(\frac{1}{1+w}\right)\left(\frac{c}{wa}+1+\ln(b)+v\right)A_x(a,b,v,c,w)$$

$$= wa\left(\frac{1}{1+w}\right)\left(\frac{c}{wa}+1+\ln(b)+v\right)\times \exp\left[\left(\frac{1}{1+w}\right)\left(\frac{c}{wa}+1+\ln(b)+\frac{v}{2}\right)-\frac{c}{wa}\right]$$

(10)

Given w, the value of c that maximizes expected stationary yield is

$$\tilde{c}_{\text{MESY}}(a,b,v,w) = \big(1-(\ln(b)+v)\,w\big)a\,. \qquad (11)$$

Note that $\tilde{c}_{\text{MESY}}(a,b,v,w)$ approaches $c_{\text{MSY}}(a,b,w)$ as v approaches zero. Also, in the special case where w=0, the solution simplifies to $\tilde{c}_{\text{MESY}}(a,b,v,0) = c_{\text{MSY}}(a,b,0) = a$ regardless of the value of v. Generally, then, the stochastic equivalent of an MSY control rule (without considering parameter uncertainty) is given by the MESY control rule

$$\tilde{h}_{\text{MESY}}(x|a,b,v,w) = \tilde{c}_{\text{MESY}}(a,b,v,w) + wa\ln(x)\,.$$

Examples of MESY control rules are shown in Figure 3. As shown previously in Figure 2, if the rebuilding level is set equal to b, an MSY control rule (i.e., a MESY control rule with v=0) always has a threshold stock size equal to b. As shown in Figure 3, however, a MESY control rule with v>0 will always have a threshold stock size less than b except in the special case where w=0. The distance between the threshold stock size and b increases monotonically with both w (seen by comparing curves *within* a particular panel of Figure 3) and v (seen by comparing curves *between* panels of Figure 3). The direct relationship between the difference b-x_{thr} and w is consistent with the fact that higher values of w imply greater cutbacks in the harvest rate as stock size falls, meaning that acceptable rates of recovery can be achieved from lower stock sizes. The direct relationship between the difference b-x_{thr} and v is consistent with the fact that natural variability is the factor that enables b to diverge from x_{thr} in the first place (i.e., in the Stage 1 case, a stock harvested under an MSY control rule will never recover to x=b in finite time).

When the right-hand side of Equation (11) is substituted for c in Equation (10), the expected value of stationary yield becomes

$$\text{MESY}(a,b,v,w) = a\,b\,e^{\left(1-\frac{1}{2(1+w)}\right)v}\,.$$

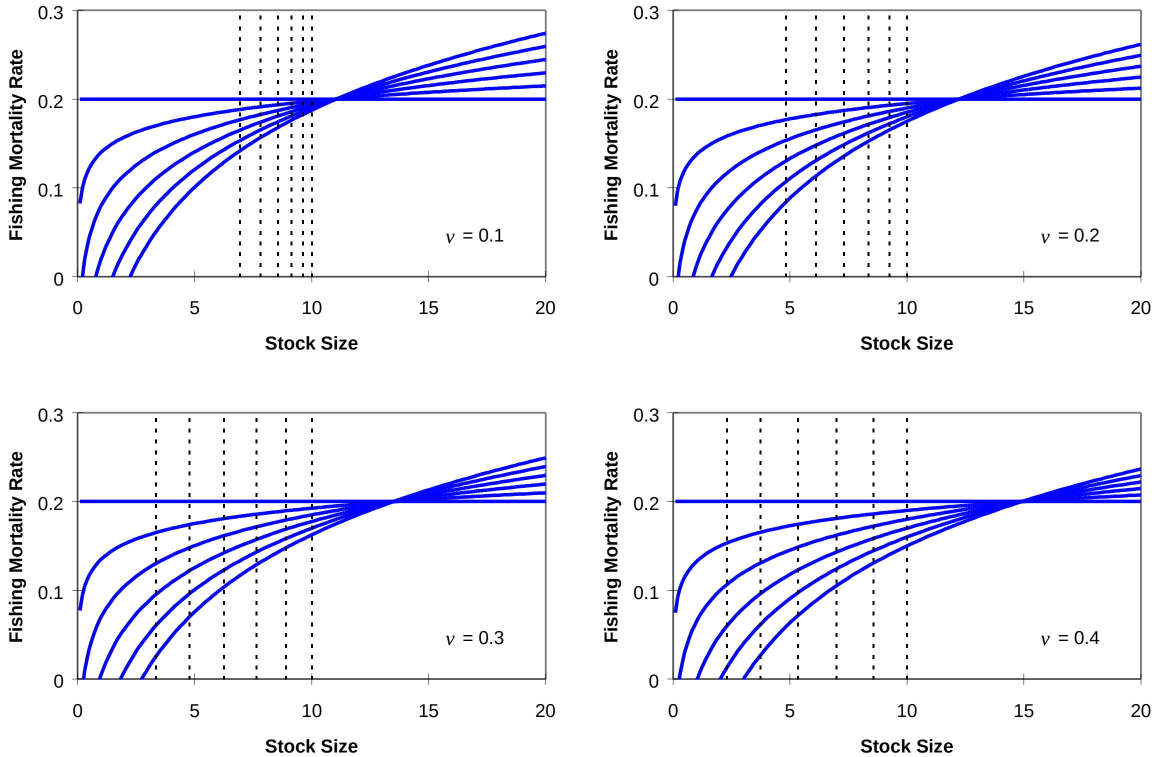

Figure 3. Example MESY control rules (solid curves) and associated thresholds (vertical dotted lines). In each panel, as *w* increases (with *c* implicit), the slope of the control rule increases and the threshold moves left.

Unlike the deterministic case where MSY was independent of the control parameter *d*, MESY does depend on the value of *d*, through the latter's dependence on *w*. The exponent in the above equation reaches a minimum of *v*/2 when *w*=0 and a maximum of *v* as *w* approaches infinity.

When $c = \tilde{c}_{\mathrm{MESY}}(a,b,v,w)$, the standard deviation of stationary yield can be written

$$\mathrm{SDSY}(a,b,v,w) =$$

$$abe^{v}\sqrt{1+\left(\frac{2+w}{1+w}\right)w\,v+\left(\frac{w}{1+w}\right)^{2}v^{2}-e^{-\frac{v}{1+w}}}\,.$$

The term under the square root symbol reaches a minimum of $1\text{-}e^{-v}$ when *w*=0 and increases without limit as *w* approaches infinity. MESY (expressed as a proportionate increase over MSY) and SDSY are plotted for *a*=*b*=1 and several values of *v* and *w* in Figure 4.

In managing a fishery, suppose that any increase in MESY were viewed as a desirable result (all other things being equal), and that likewise any decrease in SDSY were viewed as a desirable result (all other things being equal). Because both MESY and SDSY increase monotonically but nonlinearly with *w*, it may be possible to find an optimal value for *w* depending on the preference associated with a unit increase in MESY relative to the preference associated with a unit decrease in SDSY. For example, suppose that the goal was to choose the value of the control parameter *w* so as to maximize the following objective function, which uses the parameter *z* to form a linear combination of MESY and (negative) SDSY:

$$q(v,w)=\frac{z\,\mathrm{MESY}(a,b,v,w)-\mathrm{SDSY}(a,b,v,w)}{z\,\mathrm{MESY}(a,b,v,0)-\mathrm{SDSY}(a,b,v,0)}-1$$

$$=\frac{z\sqrt{e^{-\frac{v}{1+w}}}-\sqrt{1+\left(\frac{2+w}{1+w}\right)vw+\left(\frac{w}{1+w}\right)^{2}v^{2}-e^{-\frac{v}{1+w}}}}{z\sqrt{e^{-v}}-\sqrt{1-e^{-v}}}-1$$

The above equation is scaled so that *q*(*v*,0)=0. The parameter *z* represents the amount by which a unit increase in MESY is preferred relative to a unit decrease in SDSY. This objective function is plotted for several values of *v* and *z* in Figure 5.

While it is not possible to obtain a closed-form solution for the value of *w* that maximizes *q*, it is possible to derive the value of *z* for which a particular value of *w* would be optimal, given *v*:

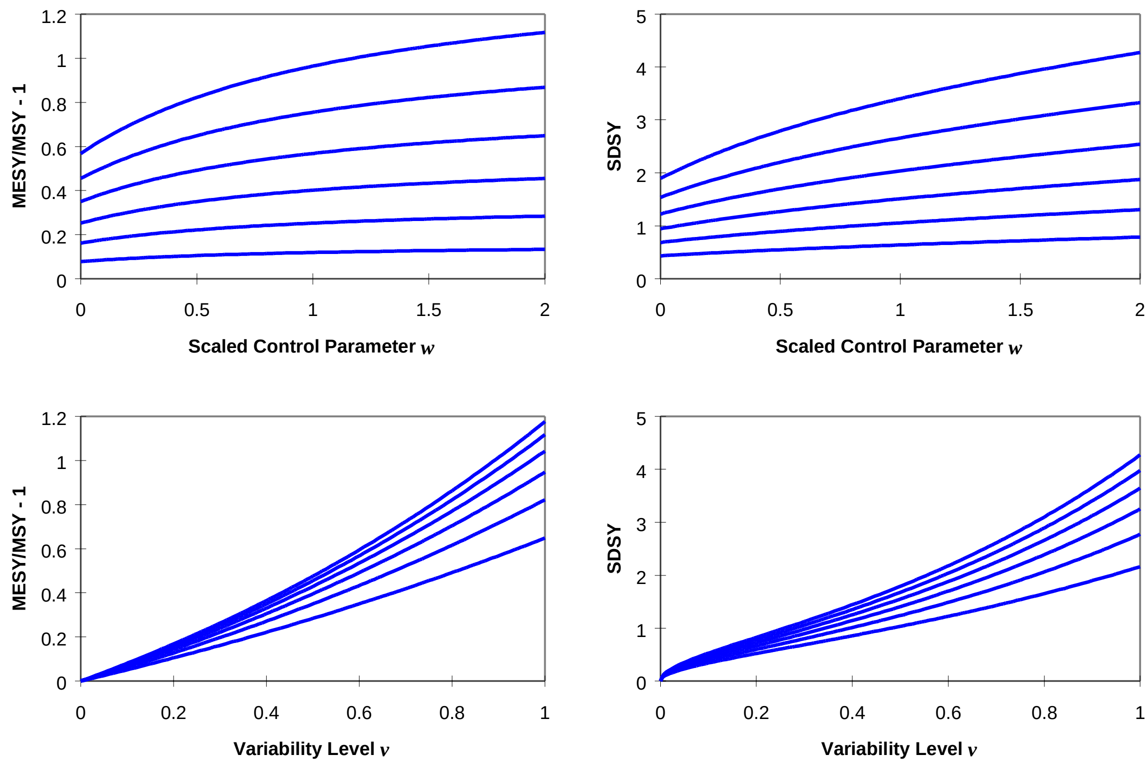

Figure 4. Profiles of MESY and SDSY. In the upper panels, higher curves correspond to higher values of variability level *v*. In the lower panels, higher curves correspond to higher values of control parameter *w*.

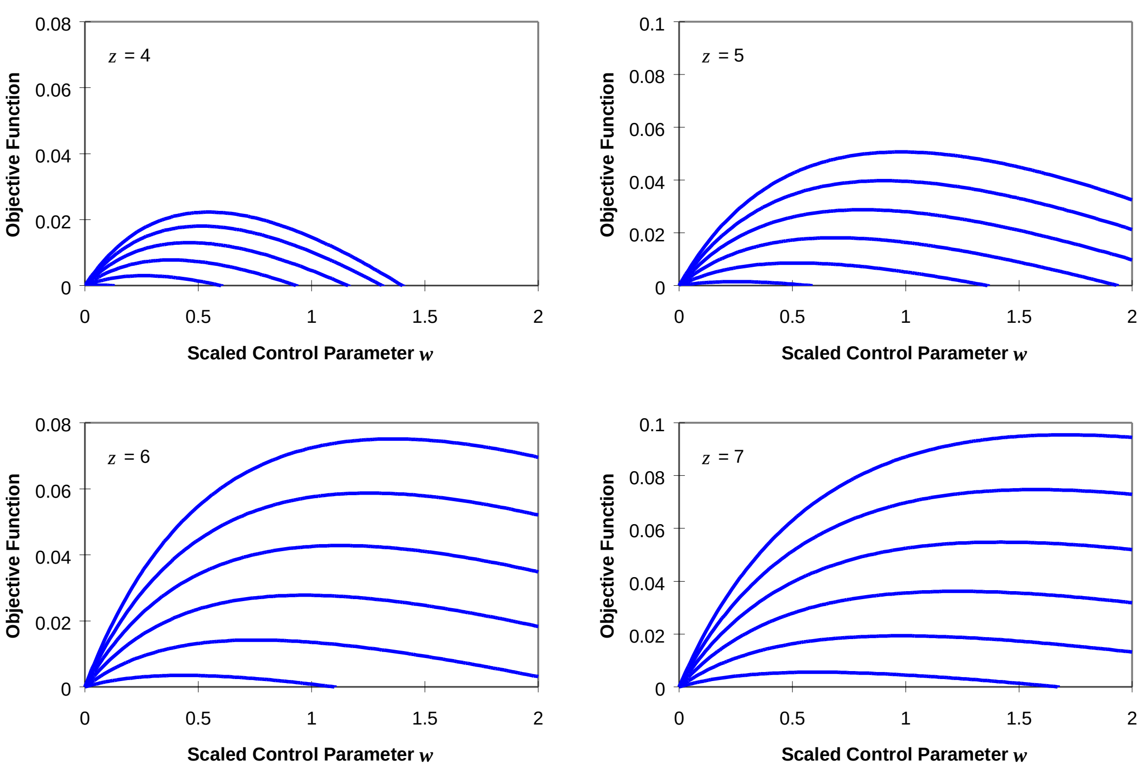

Figure 5. Examples of objective functions used to evaluate tradeoffs between mean and standard deviation of yield. In each panel, higher curves correspond to higher values of *v*. As *z* or *v* increases, maxima shift right.

$z(w_{\text{opt}}\,|\,v)=$

$$\frac{\left(w_{\text{opt}}^3+3w_{\text{opt}}^2+(4+2v)\,w_{\text{opt}}+2\right)e^{\frac{v}{1+w_{\text{opt}}}}-(1+w_{\text{opt}})}{\sqrt{\left(w_{\text{opt}}^2+(3+v)w_{\text{opt}}+2\right)vw_{\text{opt}}\,e^{\frac{v}{1+w_{\text{opt}}}}+(1+w_{\text{opt}})^2\left(e^{\frac{v}{1+w_{\text{opt}}}}-1\right)}}$$

The above relationship is plotted for several values of w_{opt} in Figure 6. Note that a positive value of w is never optimal when $z<2\sqrt{2}\approx 2.8$. Also, the value of w that maximizes q can vary considerably with v or z. For example, w=0.260 is optimal when v=0.2 and z=4, but increasing v to a value of 0.6 (with z held constant at 4) more than doubles the optimal value of w (0.546). Alternatively, increasing z to a value of 6 (with v held constant at 0.2) nearly triples the optimal value of w (0.756).

Stage 3: Incorporating Parameter Uncertainty

In General

All of the above assumes that the true values of $\boldsymbol{\chi}$ and $\boldsymbol{\psi}$ are known. When uncertainty exists regarding the true values of these parameters, additional complications arise. Many of these relate to the objective of management under a precautionary approach: Exactly what is being maximized, and how does the answer to this question differ between limit control rules and target control rules? One way to address this question is to view the distinction between limit and target control rules as a distinction between levels of relative risk aversion in a decision-theoretic framework. For example, a limit control rule might be defined by the decision-theoretic optimum derived under a risk-neutral stance, while a target control rule might be defined by the decision-theoretic optimum derived under a risk-averse stance. A simple way to characterize this difference is as follows: the risk-neutral solution maximizes the expectation of stationary yield (MESY, pronounced “mezzy”), while the risk-averse solution maximizes the expectation of log stationary yield (MELSY, pronounced “melzy”). Such use of a logarithmic loss (or utility) function in developing harvest strategies has been advocated or analyzed by Gleit (1978), Lewis (1981, 1982), Mendelssohn (1982), Opaluch and Bockstael (1984), Ruppert et al. (1984, 1985), Deriso (1985), Walters (1987), Walters and Ludwig (1987), Getz and Haight (1989), Hightower and Lenarz (1989), Hightower (1990), Parma (1990), Parma and Deriso (1990), and Thompson (1992).

Maximizing the expectation of log stationary yield is formally equivalent to maximizing the geometric mean of stationary yield. Just as the conditional arithmetic mean was defined above as a function of the parameters $\boldsymbol{\chi}$, $\boldsymbol{\psi}$, and $\boldsymbol{\omega}$, the conditional geometric mean of the stationary distribution of yield is defined (if it exists) as

$$G_y(\boldsymbol{\chi},\boldsymbol{\psi},\boldsymbol{\omega})=\exp\left(\int_0^\infty \ln\left(y(x\,|\,\boldsymbol{\omega})\right)p_x^*(x\,|\,\boldsymbol{\chi},\boldsymbol{\psi},\boldsymbol{\omega})\,\mathrm{d}\,x\right).$$

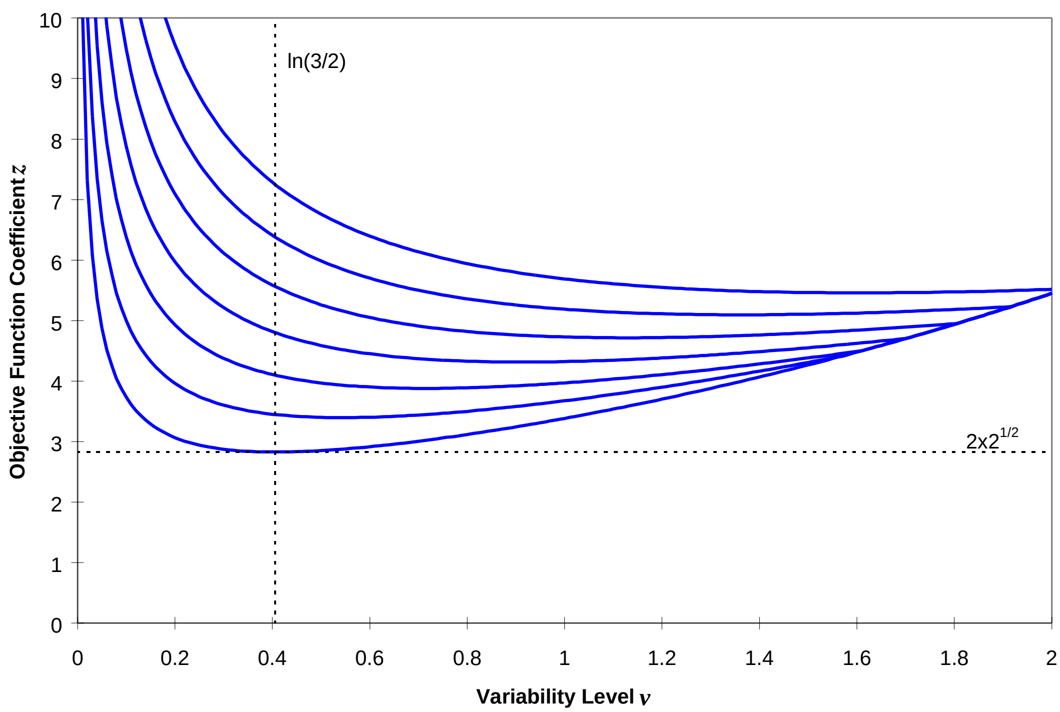

Figure 6. Value of z at which a specified value of w is optimal, given v. Beginning with the lowest curve and moving upward, curves correspond to optimal w values of 0, 0.25, 0.50, 0.75, 1.00, 1.25, and 1.50.

Likewise, in a manner analogous to that used to develop the conditional MESY solution, $G_y(\boldsymbol{\chi}, \boldsymbol{\psi}, \boldsymbol{\omega})$ can be rewritten as $G_y(c \mid \boldsymbol{\chi}, \boldsymbol{\psi}, \boldsymbol{\omega}_{(i)})$ and then maximized with respect to c, giving the control parameter value associated with the maximum expected log stationary yield, conditional on $\boldsymbol{\chi}$, $\boldsymbol{\psi}$, and $\boldsymbol{\omega}_{(i)}$:

$$\frac{\mathrm{d}\, G_y(c \mid \boldsymbol{\chi}, \boldsymbol{\psi}, \boldsymbol{\omega}_{(i)})}{\mathrm{d}\, c} \Big|_{c=\tilde{c}_{\mathrm{MELSY}}(\boldsymbol{\chi},\boldsymbol{\psi},\boldsymbol{\omega}_{(i)})} = 0 \; .$$

However, when the values of $\boldsymbol{\chi}$ and $\boldsymbol{\omega}$ are uncertain, maximization of the mean (either arithmetic or geometric) of the conditional pdf is not particularly helpful by itself, as the solution is a function of parameters whose values are unknown. Rather, it is the moments of the *marginal* pdf that are of interest. For example, the arithmetic mean of the marginal pdf is defined as

$$\bar{A}_y(\boldsymbol{\omega}) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} A_y(\boldsymbol{\chi}, \boldsymbol{\psi}, \boldsymbol{\omega})\, \mathrm{p}_{\boldsymbol{\chi},\boldsymbol{\psi}}(\boldsymbol{\chi}, \boldsymbol{\psi})\, \mathrm{d}\chi_1 \ldots \mathrm{d}\chi_m\, \mathrm{d}\psi_1 \ldots \mathrm{d}\psi_n .$$

Rewriting the above as $\bar{A}_y(c \mid \boldsymbol{\omega}_{(i)})$ and maximizing with respect to c gives $\bar{c}_{\mathrm{MESY}}(\boldsymbol{\omega}_{(i)})$; that is,

$$\frac{\mathrm{d}\, \bar{A}_y(c \mid \boldsymbol{\omega}_{(i)})}{\mathrm{d}\, c} \Big|_{c=c_{\mathrm{MESY}}(\boldsymbol{\omega}_{(i)})} = 0 .$$

The above derivation involves two operations: integration and differentiation. The order in which these two are performed can make a difference (though perhaps not always). In the above, integration precedes differentiation. In other words, the arithmetic mean of the marginal distribution of stationary yield is computed conditionally on c, then c is chosen so as to maximize this expectation. An alternative approach would be to choose the value of c that maximizes expected stationary yield conditional on $\boldsymbol{\chi}$, $\boldsymbol{\psi}$, and $\boldsymbol{\omega}_{(i)}$, and then compute the expectation of this value. This is accomplished by multiplying $\tilde{c}_{\mathrm{MESY}}(\boldsymbol{\chi}, \boldsymbol{\psi}, \boldsymbol{\omega}_{(i)})$ by $p_{\boldsymbol{\chi},\boldsymbol{\psi}}(\boldsymbol{\chi}, \boldsymbol{\psi})$ and integrating over the elements of $\boldsymbol{\chi}$ and $\boldsymbol{\psi}$, giving

$$\bar{c}_{\mathrm{MESY}}(\boldsymbol{\omega}_{(i)}) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \tilde{c}_{\mathrm{MESY}}(\boldsymbol{\chi}, \boldsymbol{\psi}, \boldsymbol{\omega}_{(i)})\; p_{\boldsymbol{\chi},\boldsymbol{\psi}}(\boldsymbol{\chi}, \boldsymbol{\psi})\, \mathrm{d}\chi_1 \ldots \mathrm{d}\chi_m\, \mathrm{d}\psi_1 \ldots \mathrm{d}\psi_n \; .$$

The same procedure can be followed for the geometric mean. The geometric mean of the marginal pdf is defined as

$$\bar{G}_y(\boldsymbol{\omega}) = \exp\left(\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \ln\big(G_y(\boldsymbol{\chi}, \boldsymbol{\psi}, \boldsymbol{\omega})\big)\; p_{\boldsymbol{\chi},\boldsymbol{\psi}}(\boldsymbol{\chi}, \boldsymbol{\psi})\, \mathrm{d}\chi_1 \ldots \mathrm{d}\chi_m\, \mathrm{d}\psi_1 \ldots \mathrm{d}\psi_n \right) .$$

Rewriting the above as $\bar{G}_y(c \mid \boldsymbol{\omega}_{(i)})$ and maximizing with respect to c gives $c_{\mathrm{MELSY}}(\boldsymbol{\omega}_{(i)})$; that is,

$$\frac{\mathrm{d}\, \bar{G}_y(c \mid \boldsymbol{\omega}_{(i)})}{\mathrm{d}\, c} \Big|_{c=c_{\mathrm{MELSY}}(\boldsymbol{\omega}_{(i)})} = 0 ,$$

while multiplying $\tilde{c}_{\mathrm{MELSY}}(\boldsymbol{\chi}, \boldsymbol{\psi}, \boldsymbol{\omega}_{(i)})$ by $p_{\boldsymbol{\chi},\boldsymbol{\psi}}(\boldsymbol{\chi}, \boldsymbol{\psi})$ and integrating over the elements of of $\boldsymbol{\chi}$ and $\boldsymbol{\psi}$ gives

$$\bar{c}_{\mathrm{MELSY}}(\boldsymbol{\omega}_{(i)}) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \tilde{c}_{\mathrm{MELSY}}(\boldsymbol{\chi}, \boldsymbol{\psi}, \boldsymbol{\omega}_{(i)})\; p_{\boldsymbol{\chi},\boldsymbol{\psi}}(\boldsymbol{\chi}, \boldsymbol{\psi})\, \mathrm{d}\chi_1 \ldots \mathrm{d}\chi_m\, \mathrm{d}\psi_1 \ldots \mathrm{d}\psi_n \; .$$

Thus, there are a number of alternative ways to proceed. In either the MESY case or the MELSY case, at least three solutions can be envisioned: 1) considering uncertainty due to natural variability only, solve for the optimum value of c as a function of the parameters $\boldsymbol{\chi}$, $\boldsymbol{\psi}$, $\boldsymbol{\omega}_{(i)}$, then evaluate that solution at the "best" estimate of those parameters (the "solve-then-evaluate" method); 2) considering uncertainty due to natural variability only, solve for the optimum value of c as a function of the parameters $\boldsymbol{\chi}$, $\boldsymbol{\psi}$, $\boldsymbol{\omega}_{(i)}$, then take the expectation of that solution over the parameters $\boldsymbol{\chi}$ and $\boldsymbol{\psi}$ (the "solve-then-integrate" method); and 3) considering both natural variability and parameter uncertainty, solve for the optimum value of c (the "integrate-then-solve" method). These three solutions are summarized in the table below:

Attitude toward risk	Solution technique	Solution notation
risk neutral	solve-then-evaluate	$\tilde{c}_{\mathrm{MESY}}(\boldsymbol{\chi}, \boldsymbol{\psi}, \boldsymbol{\omega}_{(i)})$
risk neutral	solve-then-integrate	$\bar{c}_{\mathrm{MESY}}(\boldsymbol{\omega}_{(i)})$
risk neutral	integrate-then-solve	$c_{\mathrm{MESY}}(\boldsymbol{\omega}_{(i)})$
risk averse	solve-then-evaluate	$\tilde{c}_{\mathrm{MELSY}}(\boldsymbol{\chi}, \boldsymbol{\psi}, \boldsymbol{\omega}_{(i)})$
risk averse	solve-then-integrate	$\bar{c}_{\mathrm{MELSY}}(\boldsymbol{\omega}_{(i)})$
risk averse	integrate-then-solve	$c_{\mathrm{MELSY}}(\boldsymbol{\omega}_{(i)})$

For Example

In Stage 2, the quantity w (defined as $w \equiv d/a$) was a constant. To retain the interpretation of w as a constant in Stage 3, it will prove convenient at this point to redefine $w \equiv d/A_a$ and to reparametrize the model accordingly. Thus, wherever w appears as a parameter in a previous equation, it can be replaced with the quantity $A_a w/a$. Use of the redefined parameter w renders many quantities of interest in this model proportional to A_a.

A general solution for $c_{\mathrm{MESY}}(w)$cannot be obtained, because any particular solution will depend on the form of the joint pdf of a, b, and v. However, because $\tilde{c}_{\mathrm{MESY}}(a, b, v, w)$ is linear in a, ln(b), and v [Equation (11)], the following solution for $\bar{c}_{\mathrm{MESY}}(w)$ will be independent of the form of the joint pdf of a, b, and v:

$$\bar{c}_{\mathrm{MESY}}(w) = \big(1 - \big(\ln(G_b) + A_v\big)\, w\big)\, A_a \qquad (12)$$

Obtaining general solutions for MELSY is even more difficult than in the MESY case. For one thing, the fact that the logarithmic control rule forces yield to equal zero at x=exp(-c/d) means that $\overline{G}_y(a,b,v,c,w)$ does not exist except when w=0. For purposes of illustration, however, an exact solution for a quantity closely related to $c_{\mathrm{MELSY}}(w)$ can be obtained if a, b, and v are assumed to be independent and if particular functional forms are chosen for their respective pdfs. Specifically, let $p_a(a)$ follow a 3-parameter F distribution with scale parameter d, let $p_b(b)$ follow a lognormal distribution, and let $p_v(v)$ follow an inverse Gaussian distribution (Appendix).

For any positive random variable, the ratio of the harmonic mean to the arithmetic mean may be viewed as a measure of the degree of certainty surrounding the value of that variable. This ratio ranges from a lower bound no less than zero, representing complete uncertainty, to an upper bound no greater than unity, representing complete certainty (e.g., Mitrinović et al. 1993). For the particular distributional forms assumed here, the ratios of harmonic to arithmetic means may be expressed in terms of the coefficient of variation (CV) as follows (the harmonic and arithmetic means are also given as functions of their respective distributional parameters in the Appendix):

$$\frac{H_a}{A_a} = \frac{1 + w + (1-w)\,\mathrm{CV}_a^2}{1 + w + 2\,\mathrm{CV}_a^2} \;,$$

$$\frac{H_b}{A_b} = \frac{1}{1+\mathrm{CV}_b^2} \;,$$

$$\frac{H_v}{A_v} = \frac{1}{1+\mathrm{CV}_v^2} \;.$$

Given the assumption that a follows an F distribution with scale parameter d, the quantity $u \equiv a/(a+d)$ is beta-distributed with arithmetic mean

$$A_u = \frac{\dfrac{H_a}{A_a} + w}{\dfrac{H_a}{A_a} + 2w + w^2} \;.$$

Then, a quantity closely related to $c_{\mathrm{MELSY}}(w)$ can be written (Appendix) as

$$\hat{c}_{\mathrm{MELSY}}(w) = \left(k_a - \left(\ln(G_b k_b) + A_v k_v \right) w \right) A_a, \quad (13)$$

where

$$k_a = \left(\frac{H_a}{A_a} + w \right)\left(\frac{1}{1+w} \right),$$

$$k_b = \left(\frac{H_b}{A_b} \right)^{-A_u},$$

$$k_v = \left(1 - A_v \left[\left(\frac{H_v}{A_v} \right)^{-1} - 1 \right] A_u \right)^{-\frac{1}{2}} .$$

For all practical purposes, the adjustment factors k_a, k_b, and k_v vary directly with the ratios H_a/A_a, H_b/A_b, and H_v/A_v, respectively, so that an increase in uncertainty regarding any of the parameters results in a downward shift in the control rule. Examples of limit control rules (using Equation (12)) and target control rules (using Equation (13)) are shown in Figure 7 for four values of w, in Figure 8 for four values of $H_a/A_a = H_b/A_b = H_v/A_v$, and in Figure 9 for four values of A_v (because the axes in Figures 7-9 are scaled relative to A_a and G_b, the curves are independent of these two parameters). The upper left panels of Figures 7-9 are all identical, giving a common point of reference against which to contrast results associated with different parameter values. In each of these figures, the control rules developed under this model are contrasted with the existing control rules for "Tier 1" of the harvest policy established in 1996 for Alaska groundfish (U.S. National Marine Fisheries Service 1996).

Discussion

Harvest control rules provide a tractable and heuristic means of comparing alternative fishery management strategies. They can be analyzed in the context of a wide variety of models, ranging from simple deterministic models with known parameter values to complex stochastic models with uncertain parameter values. Moving from the classical one-parameter control rules (e.g., constant fishing mortality, constant escapement) to a two-parameter control rule such as the logarithmic form considered in the example model here can sometimes render comparisons between the former more meaningful by framing them as special cases along a continuum of possible strategies rather than as conceptually unrelated policies. More elaborate functional forms, in which various two-parameter control rules might emerge as special cases, can also be imagined. Generally, the ideal level of complexity to build into a harvest control rule, as well as the appropriate number of parameters to be left free therein, remain open questions. Relative to a full optimal control solution, some degree of optimality may be sacrificed whenever the functional form of the control rule is constrained *a priori*. However, the sacrifice may be slight. For example, in the deterministic Gordon-Schaefer model considered by Clark (1976), the optimal control solution consisted of a one-parameter constant escapement policy. Even when more complicated stochastic models are used, the difference between a full optimal control solution and a fixed-form optimization can be negligible (e.g., Mendelssohn 1980, Horwood 1993).

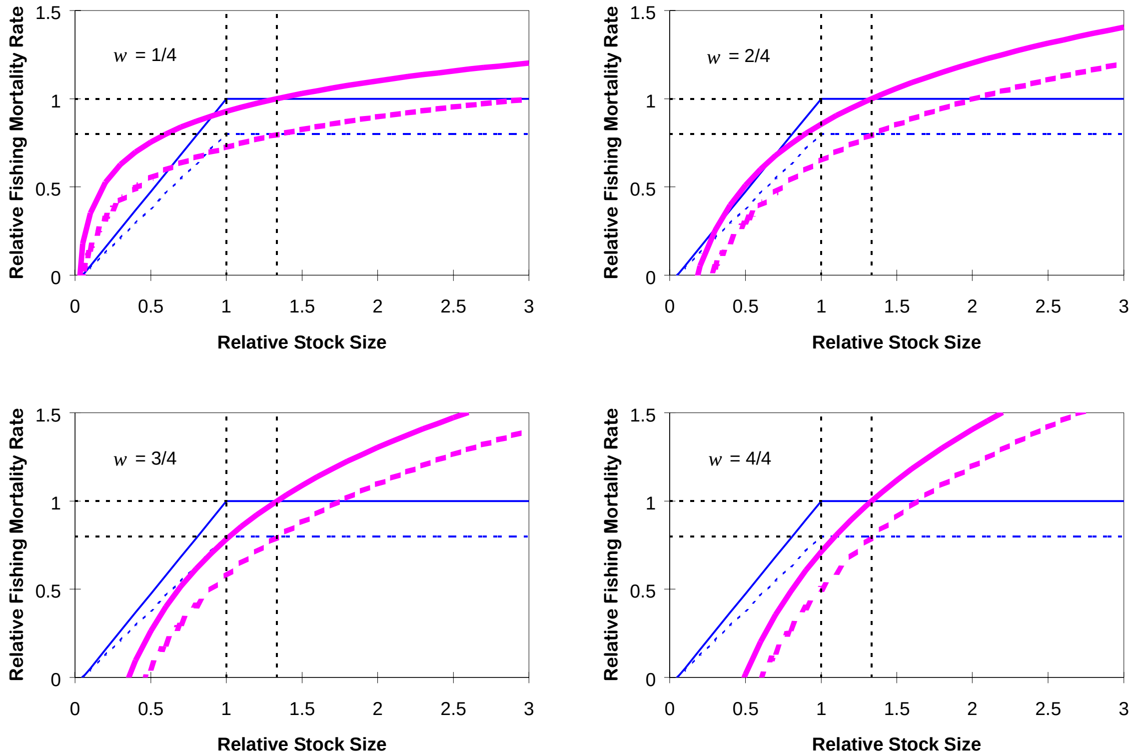

Figure 7. Limit (solid) and target (dashed) control rules in the example model (thick) and Alaska groundfish policy (thin). Horizontal and vertical dotted lines depict additional reference points.

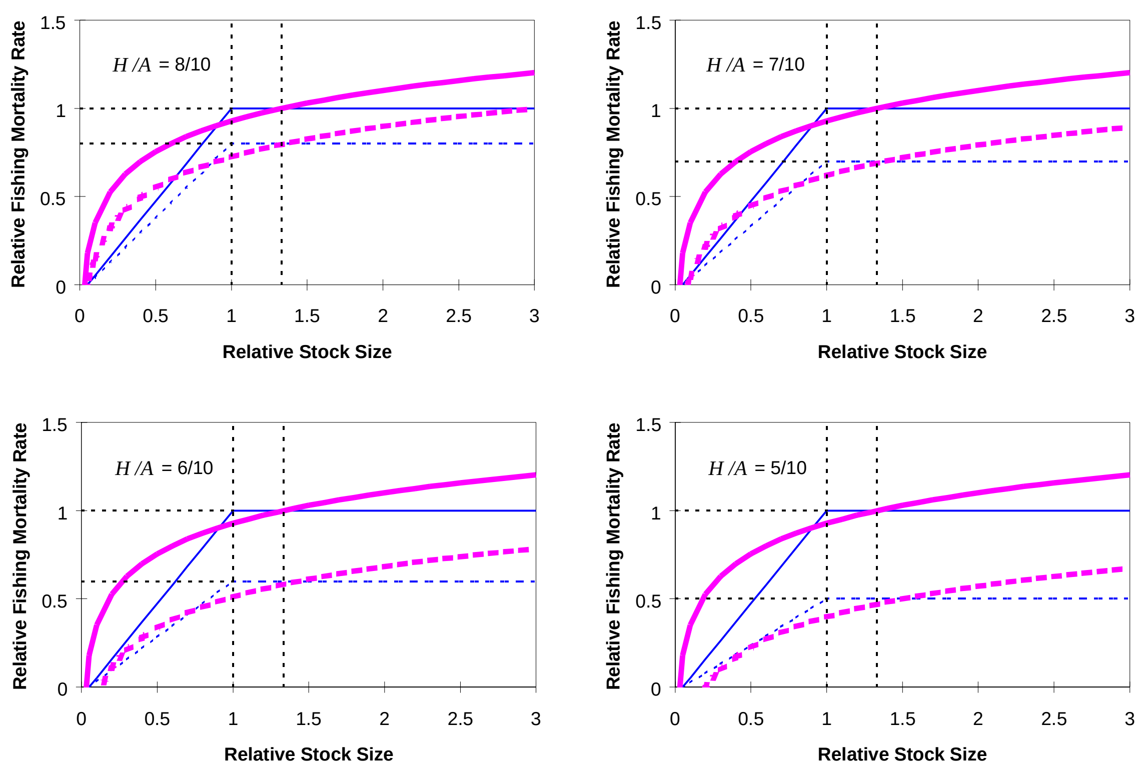

Figure 8. Limit (solid) and target (dashed) control rules in the example model (thick) and Alaska groundfish policy (thin). Horizontal and vertical dotted lines depict additional reference points.

For the most part, it has been assumed here that optimization of the control rule is confined to a single control parameter (c, in the case of the example model). This is a convenient restriction, but not a necessary one. For instance, in the Stage 2 example model considered here, it is possible to maximize expected stationary yield with respect to both c and d. This results in a strategy of the constant escapement type, confirming the conclusion of Reed (1978) and others that a constant escapement policy dominates the other classical one-parameter control rules when the objective is maximization of long-term average yield. However, leaving at least one control parameter (say, d) to be fixed independently of the optimization has previously been advocated (e.g., Ruppert et al. 1984) on the basis that it facilitates consideration of management objectives other than yield maximization. For instance, in the Stage 2 example model considered here, allowing d to be fixed independently means that the full range of tradeoffs between long-term average yield and the level of variability around that average can be presented (Figure 5). As in previous studies (e.g., Beddington and May 1977), the example model shows that the arithmetic mean and standard deviation of stationary yield vary together. However, the example model here goes further than previous studies in showing that this result holds true across a continuum of MSY control rules (i.e., as a function of control parameter d with c set at its conditional MESY value).

Figures 7-9 contrast the example model with the existing control rules for "Tier 1" of the harvest policy established in 1996 for Alaska groundfish (U.S. National Marine Fisheries Service 1996). The three-parameter control rules used in the Alaska groundfish policy are, in fact, based partly on a special case of the example model. Specifically, the horizontal portions of those control rules correspond to the special cases of Equations (12) and (13) in which w=0. When w=0, $\bar{c}_{\mathrm{MESY}}$ and c_{MELSY} (or $\hat{c}_{\mathrm{MELSY}}$) are simply the arithmetic and harmonic means, respectively, of the marginal distribution of a. Interestingly, this result holds regardless of the functional form of the joint distribution of a, b, and s. In contrast, the general ($w \geq 0$) form of Equation (13) depends on several assumptions regarding the joint distribution of a, b, and s.

As Figures 7-9 show, the logarithmic control rule used in the example model can be implemented in a manner that satisfies the requirements of a precautionary approach specified by the U.S. Department of Commerce (1998): 1) target harvest rates are less than limit harvest rates, 2) harvest rates at low stock sizes are less than harvest rates at high stock sizes, and 3) the buffer between limit and target harvest rates widens as uncertainty regarding a stock's size or productive capacity increases. The use of a logarithmic control rule (with w>0) automatically satisfies the second requirement, whereas satisfaction of the first and third requirements

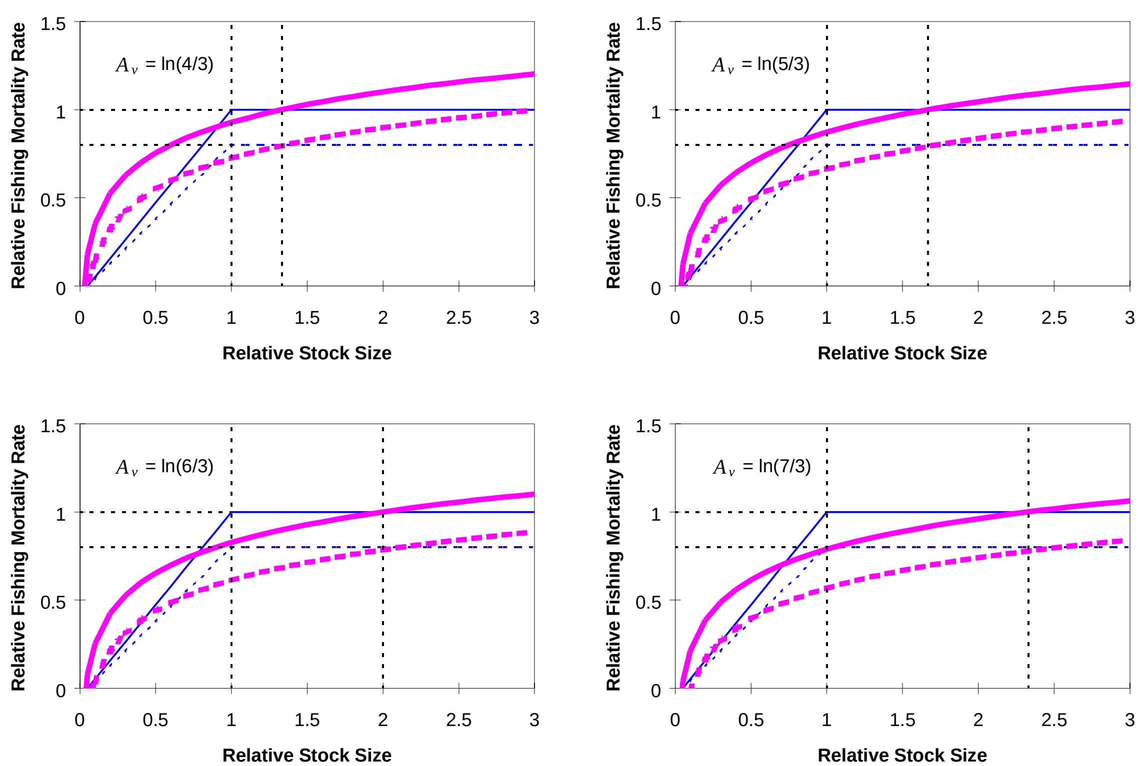

Figure 9. Limit (solid) and target (dashed) control rules in the example model (thick) and Alaska groundfish policy (thin). Horizontal and vertical dotted lines depict additional reference points.

is achieved by basing the limit control rule on a *risk-neutral* optimization and the target control rule on a *risk-averse* optimization. The Alaska groundfish policy also satisfies these three requirements, with the exception that the size of the buffer increases directly with uncertainty regarding productive capacity only (not stock size).

It may also be noted that the limit control rule shown for the example model in Figures 7-9 qualifies as an MSY control rule, whereas the limit control rule used in the Alaska groundfish policy does not. The failure of the limit control rule used in the Alaska groundfish policy to qualify as an MSY control rule is due to the fact that the presence of the descending limb was not considered in the process of setting the height of the horizontal limb. That is, in setting the height of the horizontal limb, the optimization was conditional on the assumption that a constant fishing mortality policy would apply, whereas in fact such a policy applies only when the stock is above its MSY level.

In Stages 1 and 2, it was assumed that estimates of the biological parameters a, b, and s (or v) are obtainable. In Stage 3, it was assumed that pdfs of these parameters are obtainable. In practice, obtaining these estimates or distributions will typically be a non-trivial exercise. However, the functional form of the example model described here is particularly amenable to this task. Thompson (1998) showed how a log transformation of this model satisfies the assumptions of the Kalman filter (e.g., Harvey 1990) exactly, meaning that either maximum likelihood or Bayesian methods can be used in a straightforward manner to obtain parameter estimates or posterior distributions of parameters (if maximum likelihood is used, distributions could be obtained by appealing to the asymptotic normality of the parameter estimates). The model is sufficiently simple, in fact, that the maximum likelihood estimate of the deterministic carrying capacity ($=be$) can be written in closed form.

The subject of rebuilding depleted stocks was considered for the Stage 1 and Stage 2 cases, but not for the Stage 3 case. A Stage 3 treatment should not prove too problematic, however, insofar as computing the geometric or arithmetic mean of Equation (6) for the case where the values of a and b are uncertain does not appear to pose any special difficulty (note that the natural variability parameter s does not enter into Equation (6)). Despite the omission of a Stage 3 treatment of rebuilding, the results obtained under Stages 1 and 2 in the example model offer some interesting insights on their own. For example, suppose that the goal of a rebuilding program is to return a depleted stock to its deterministic MSY stock size b. In this case, the Stage 1 example model indicates that the threshold stock size prescribed by any MSY control rule will also be equal to b regardless of the allowable time frame for rebuilding. Thus, under Stage 1 conditions, anytime a stock falls below its deterministic MSY stock size, it will be impossible to rebuild to the deterministic MSY level in finite time while fishing according to any MSY control rule. In the Stage 2 example model, however, the conclusions are different. Specifically, if the geometric mean of the transition distribution is used to define the threshold stock size, the threshold stock size prescribed by any MESY control rule with d>0 and s>0 will always be less than b regardless of the allowable time frame for rebuilding. Thus, under Stage 2 conditions, it is possible for a stock to fall below its deterministic MSY stock size to some extent and still rebuild to the deterministic MSY level within an allowable time frame while fishing according to a given MESY control rule. The difference in conclusions reached under Stages 1 and 2 in this regard is due to the fact that a Stage 1 MSY control rule evaluated at the point $x=b$ always gives a harvest rate equal to a, whereas a Stage 2 MESY control rule evaluated at the same point always gives a harvest rate less than a so long as d>0 and s>0. However, under a MESY control rule with d=0 (i.e., a constant fishing mortality policy), even the Stage 2 example model prescribes a threshold stock size equal to b.

Another aspect of rebuilding that was not addressed here is the question of whether rebuilding should be viewed primarily in terms of stock size x or in terms of rebuilding time t. In other words, is it more important to consider the probability that the stock size will exceed x_{reb} at time t_{reb}, or the probability that the time needed for the stock size to exceed x_{reb} will be greater than t_{reb}? The two approaches are not equivalent (e.g., Dennis et al. 1991).

In conclusion, some caveats are probably appropriate. First, the logarithmic control rule used in the example model exhibits some features that may require getting used to. For example, one must either interpret the control rule as exhibiting a discontinuity at the point where it crosses the x axis (making the mathematics more complicated), or be prepared to accept (as an approximation, at least) the idea of a small negative "yield" at sufficiently low stock sizes. Also, the fact that the control rule has no finite upper bound may not be appealing to some. Second, the results pertaining to the example model may not extend to other models. For instance, a discrete rather than a continuous representation of stock dynamics, other functional forms for Equation (1), or other interpretations of the stochastic differential (Equation (7); for example, Ricciardi 1977) could alter the conclusions either quantitatively or qualitatively. Finally, the derivation of the MELSY solution presented in Equation (13) requires some strong assumptions. For instance, the assumption that the parameters a and v are independent is problematic unless a and s happen to vary together in a particular manner. Also, the form assumed for the pdf of a implies that, for given values of H_a and A_a, the coefficient of variation changes with the choice of w, which is probably an undesirable property.

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Appendix: Derivation of a MELSY Solution

As discussed in the main text, one type of MELSY solution is achieved by maximizing the geometric mean of the marginal distribution of stationary yield, which can be written as

$$\bar{G}_y(c,d) = \exp\left[\begin{array}{l}\int_0^\infty \int_0^\infty \int_0^\infty \left(\int_0^\infty \ln\left(y(x\,|\,c\,,d\,)\right) p_x^*(x\,|\,a\,,b\,,s\,,c\,,d)\,\mathrm{d}x\right) \\ p_{a,b,s}(a\,,b\,,s)\,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}s\end{array}\right].$$

Unfortunately, several difficulties arise. For example, if the joint distribution of the parameters $p_{a,b,s}(a,b,s)$ is left completely general, it is not possible to obtain an analytic solution except for the special case in which d=0. The following simplifying assumptions will therefore be made:

1) The model can be reparametrized by substituting $\sqrt{2av}$ for s wherever the latter occurs.

2) The parameters a, b, and v are independent, so that $p_{a,b,v}(a,b,v)=p_a(a)p_b(b)p_v(v)$.

The above assumptions imply that the solution can be written as

$$\bar{G}_y(c\,,d) = \exp\left[\begin{array}{l}\int_0^\infty \int_0^\infty \int_0^\infty \left(\int_0^\infty \ln\left(y(x\,|\,c,d)\right) p_x^*(x\,|\,a,b,v,c,d)\,\mathrm{d}x\right) \\ p_a(a)\mathrm{d}a\; p_b(b)\mathrm{d}b\; p_v(v)\mathrm{d}v\end{array}\right].$$

Next, there is a problem in that the control rule $h(x) = c + d\ln(x)$ implies that yield falls to zero at x=exp(-c/d), at which point the logarithm no longer exists. Therefore, a compromise will be made by defining, tentatively, a "quasi-geometric mean" that involves taking the logarithm *after* integrating with respect to x (rather than before):

$$\hat{G}_y(c\,,d) \equiv \exp\left[\begin{array}{l}\int_0^\infty \int_0^\infty \int_0^\infty \ln\left(\int_0^\infty y(x\,|\,c,d\,)\; p_x^*(x\,|\,a,b,v,c,d)\,\mathrm{d}x\right) \\ p_a(a)\mathrm{d}a\; p_b(b)\mathrm{d}b\; p_v(v)\mathrm{d}v\end{array}\right]$$

$$= \exp\left[\begin{array}{l}\int_0^\infty \int_0^\infty \int_0^\infty \ln\left(A_y(a\,,b\,,v\,,c\,,d\,)\right) \\ p_a(a)\mathrm{d}a\; p_b(b)\mathrm{d}b\; p_v(v)\mathrm{d}v\end{array}\right].$$

Next, there is a problem in that the form of the conditional arithmetic mean implies that yield falls to zero at b=exp(-c/d-1-v), at which point the logarithm no longer exists. Therefore, another compromise will be made by redefining the "quasi-geometric mean" so that the exponentiation occurs immediately after integrating with respect to a (rather than waiting until *all* integrations have been completed):

$$\hat{G}_y(c\,,d) \equiv \int_0^\infty \int_0^\infty \exp\left[\int_0^\infty \ln\left(A_y(a\,,b\,,v\,,c\,,d)\right) p_a(a)\mathrm{d}a\right] p_b(b)\mathrm{d}b\; p_v(v)\mathrm{d}v\ .$$

Finally, it will be assumed that $p_a(a)$, $p_b(b)$, $p_v(v)$ have particular functional forms. Specifically, the following will be assumed:

1) The uncertainty surrounding a can be described by an F distribution with scale parameter d, meaning that the uncertainty surrounding the variable u=a/(a+d) can be described by a beta distribution, that is,

$$p_a(a) = \frac{\left(\frac{a}{d}\right)^{\alpha_a - 1}\left(1 + \frac{a}{d}\right)^{-\alpha_a - \beta_a}}{d\left(\frac{\Gamma(\alpha_a)\,\Gamma(\beta_a)}{\Gamma(\alpha_a + \beta_a)}\right)}$$

or

$$p_u(u) = \frac{u^{\alpha_a - 1}(1 - u)^{\beta_a - 1}}{\frac{\Gamma(\alpha_a)\,\Gamma(\beta_a)}{\Gamma(\alpha_a + \beta_a)}}\ ,$$

where a_a and b_a are parameters. The harmonic and arithmetic means of a are given by

$$H_a = d\left(\frac{\alpha_a - 1}{\beta_a}\right)$$

and

$$A_a = d\left(\frac{\alpha_a}{\beta_a - 1}\right),$$

respectively. The ratio of H_a to A_a (i.e., the degree of certainty regarding the true value of a) is thus independent of d. The arithmetic mean of u is dependent on both the scaled control parameter w and the degree of uncertainty regarding the true value of a, as shown below:

$$A_u = \frac{\alpha_a}{\alpha_a + \beta_b} = \frac{\frac{H_a}{A_a} + w}{\frac{H_a}{A_a} + 2w + w^2}.$$

2) The uncertainty surrounding b can be described by a lognormal distribution, that is,

$$p_b(b) = \sqrt{\frac{1}{2\pi}}\left(\frac{1}{\sigma_b\, b}\right)\exp\left[-\left(\frac{1}{2}\right)\left(\frac{\ln(b) - \mu_b}{\sigma_b}\right)^2\right],$$

where μ_b and σ_b are parameters. The harmonic, geometric, and arithmetic means of b are given by

$$H_b = \exp\left(\mu_b - \frac{\sigma_b^{\,2}}{2}\right),$$

$$G_b = \exp\left(\mu_b\right),$$

and

$$A_b = \exp\left(\mu_b + \frac{\sigma_b^{\,2}}{2}\right),$$

respectively.

3) The uncertainty surrounding v can be described by an inverse Gaussian distribution, that is,

$$p_v(v) = \frac{\left(\dfrac{v}{\eta_v}\right)^{-\frac{3}{2}} \exp\left(-\left(\dfrac{\theta_v}{2}\right)\left[\left(\dfrac{v}{\eta_v}\right) + \left(\dfrac{v}{\eta_v}\right)^{-1}\right]\right)}{\eta_v e^{-\theta_v}\sqrt{\dfrac{2\pi}{\theta_v}}},$$

where η_v and θ_v are parameters. The harmonic and arithmetic means of v are given by

$$H_v = \eta_v\left(\frac{\theta_v}{\theta_v + 1}\right)$$

and

$$A_v = \eta_v,$$

respectively.

Given the above, $\hat{G}_y(c, d)$ can be written, up to a constant of proportionality, as follows:

$$\hat{G}_y(c, d) \propto \left[c + \left(1 + \mu_b + \sigma_b^2 A_u + \eta_v\sqrt{\frac{\theta_v}{\theta_v - \eta_v A_u}}\right)d\right] \times \exp\left[c\left(\frac{A_u - 1}{d}\right)\right]$$

Differentiating the above with respect to c, setting the resulting expression equal to zero, solving for c, and substituting wA_a for d gives

$$\hat{c}_{\mathrm{MELSY}}(w) = \left(k_a - \left(\ln(G_b k_b) + A_v k_v\right) w\right) A_a,$$

where

$$k_a = \left(\frac{H_a}{A_a} + w\right)\left(\frac{1}{1 + w}\right),$$

$$k_b = \left(\frac{H_b}{A_b}\right)^{-A_u},$$

$$k_v = \left(1 - A_v\left[\left(\frac{H_v}{A_v}\right)^{-1} - 1\right] A_u\right)^{-\frac{1}{2}}.$$